Determining When an Expression Is Undefined

Have you ever…

- Tried to use a calculator to divide by zero and gotten an error?

- Tried to figure out the square root of a negative number?

Expressions such as $2 \div 0$ and $\sqrt{-3}$ look like they make mathematical sense. Zeros and negative numbers are used in equations and expressions all the time. These expressions are special, though, because they can't be simplified to a real number. In these situations, the expression is **undefined**.

So far, you've simplified numerical expressions where the answer is a real number. For instance, the expression $21 - 4(3) + 0.8$ simplifies to 9.8, which is a real number. Similarly, the principal square root of 25 is 5, which is also a real number.

Certain numerical expressions don't simplify to a real number. In these cases, the answer is said to be **undefined**. There are two situations where this can happen; division by zero and square roots of negative numbers.

- **Dividing by Zero:** Why is dividing by zero undefined? The reason has to do with the related multiplication. For instance. if you divide 12 by 4, the answer is 3 because 3 times 4 is 12. However, if you divide 12 by zero, then you are asking, "What number times zero is equal to 12?" Of course, this is not possible because any number times zero is zero. That's why division by zero is undefined.

- **Square Roots of Negative Numbers:** Why is the square root of a negative number undefined? You know that the square root of 25 is 5. That's because 5 times 5 is 25. What about the square root of –25? There is no number that you can multiply by itself to get –25. That's why the square root of a negative number is undefined.

Learn It!

Undefined Expressions

Some numerical expressions are undefined. This usually happens when you are dividing a number by zero, or when you are taking the square root of a negative number.

Determine which expressions are undefined.

$$\text{A)} \sqrt{10-14} \quad \text{B)} \sqrt{-36} \quad \text{C)} \frac{4}{-3+3} \quad \text{D)} \frac{0}{12}$$

Simplify the Expression

Write the expression in simplified form if it isn't already. This makes it easier to determine if the expression is undefined.

?   **1.** What is the simplified form of expression A?

To simplify this expression, get a single number under the square root symbol.

$$\sqrt{10-14} = \sqrt{-4}$$

Determine If the Expression Is Undefined

Check for the two cases for identifying undefined expressions. If there is division by zero or the square root of a negative number, the expression is undefined.

?   **2.** Is expression A undefined?

Expression A is undefined because the square root of a negative number is undefined.

Repeat the Steps with the Other Expressions

If you're checking multiple expressions, use the same steps for each. Simplify, and check for division by zero or a square root of a negative number.

?   **3.** Simplify expression B and check if it is undefined.

Expression B is already simplified because there is a single number under the radical sign. This expression is undefined because the number under the radical sign is negative.

?   **4.** Simplify expression C and check if it is undefined.

Expression C simplifies to $\frac{4}{0}$ because $-3 + 3 = 0$.

This expression is undefined because the denominator is zero.

?   **5.** Simplify expression D and check if it is undefined.

Expression D is already simplified because there is a single number in the numerator and the denominator.

This expression is non undefined because there is not a zero in the denominator. This expression is equal to zero, because zero divided by any non-negative number is zero.

Practice It!

For each expression, write the expression in simplified form and evaluate if the expression is undefined.

★ **1.** $\dfrac{3}{5-5}$

a. Write the expression in simplified form.

b. Evaluate whether the expression is undefined. Why or why not?

★ **2.** $\sqrt{121}$

a. Write the expression in simplified form.

b. Evaluate whether the expression is undefined. Why or why not?

★ **3.** $\dfrac{6-2(3)}{6}$

a. Write the expression in simplified form.

b. Evaluate whether the expression is undefined. Why or why not?

4. $\sqrt{4(3) - 20}$

    a. Write the expression in simplified form.

    b. Evaluate whether the expression is undefined. Why or why not?

5. $\dfrac{5 + 8}{2(3 - 3)}$

    a. Write the expression in simplified form.

    b. Evaluate whether the expression is undefined. Why or why not?

6. $2 + \sqrt{2(4) - 8}$

    a. Write the expression in simplified form.

    b. Evaluate whether the expression is undefined. Why or why not?

7. $\dfrac{\sqrt{6^2 - 32}}{2^3 - 8}$

    a. Write the expression in simplified form.

    b. Evaluate whether the expression is undefined. Why or why not?

8. Think about the expression $\sqrt[4]{-16}$.

This expression would be equal to x if $x \times x \times x \times x = -16$.

Compare this expression to the square root of a negative number and the reasons why a square root of a negative number is undefined.

Is this expression undefined? Why or why not?

Check **Your Skills**

When you see this icon, you may use a calculator.

Use your knowledge of undefined numbers to answer the following questions.

1. In which of the following cases is the square root of a number undefined?

    a. When the numerator is zero and the denominator is non-zero

    b. When the denominator is positive and the numerator is zero

    c. When the number under the square root symbol is positive

    d. When the number under the square root symbol is negative

2. When is a fraction undefined?

    a. When the numerator is zero

    b. When the denominator is zero

    c. When the numerator is negative

    d. When the number denominator is negative

3. Which expressions are undefined? Select all that apply.

    ☐ $\dfrac{4-4}{4+4}$

    ☐ $\sqrt{0-7}$

    ☐ $\left(\dfrac{3}{5}\right) \div 0$

    ☐ $\sqrt{15-15}$

4. For which value of x is the expression $\dfrac{x}{x-4}$ undefined?

    a. 0

    b. -4

    c. 4

    d. -1

5. For which value of x is the expression $\sqrt{4+x}$ undefined?

    a. -4

    b. 0

    c. 4

    d. -5

> *Remember the Concept*
>
> - Divison by zero is undefined.
>
> - The square root of a negative number is undefined.

Determining When an Expression Is Undefined

Undefined Expressions

Practice It!

1a. $\frac{3}{0}$

In the denominator, $5 - 5 = 0$, so the expression simplifies to $\frac{3}{0}$.

1b. Since the denominator is zero, the expression is undefined.

2a. $\sqrt{121}$

The expression is already in simplified form because there is a single number under the square root sign.

2b. Since the number under the square root sign is positive, the expression is not undefined ($\sqrt{121} = 11$)

3a. $\frac{0}{6}$

Using the order of operations, the numerator is $6 - 2(3) = 6 - 6 = 0$. The expression simplifies to $\frac{0}{6}$.

3b. Since the denominator is not zero, the expression is not undefined ($\frac{0}{6} = 0$).

4a. $\sqrt{-8}$

Using the order of operations, the expression under the square root sign is $4(3) - 20 = 12 - 20 = -8$. The expression simplifies to $\sqrt{-8}$.

4b. Since the number under the square root sign is negative, the expression is undefined.

5a. $\frac{13}{0}$

In the numerator $5 + 8 = 13$, and in the denominator $2(3 - 3) = 2(0) = 0$, so the expression simplifies to $\frac{13}{0}$.

5b. Since the denominator is zero, the expression is undefined.

6a. 2

Using the order of operations, the expression under the square root sign is $2(4) - 8 = 8 - 8 = 0$. The square root of zero is zero. The expression simplifies to 2.

$2 + \sqrt{2(4) - 8} = 2 + \sqrt{8 - 8} = 2 + \sqrt{0} = 2$

6b. Since the number under the square root sign is not negative, the expression is not undefined.

7a. $\frac{2}{0}$

Simplify the numerator and the denominator:
$$\frac{\sqrt{6^2 - 32}}{2^3 - 8} = \frac{\sqrt{36 - 32}}{8 - 8} = \frac{\sqrt{4}}{0} = \frac{2}{0}$$

7b. Since the denominator is zero, the expression is undefined.

8. The expression is undefined. There is no number x that you can multiply by itself four times to get a negative number. Try some example negative numbers:

$-1 \times -1 \times -1 \times -1 = 1$

$-2 \times -2 \times -2 \times -2 = 16$

$-3 \times -3 \times -3 \times -3 = 81$

If the number is negative, the answer is always positive. The first two negative numbers you multiply give you a positive number. When you multiply the positive number by a negative number, you always get a negative number. When you multiply that negative result by the fourth negative number, you always get a positive result:

$-x \times -x = x^2$

$x^2 \times -x = -x^3$

$-x^3 \times -x = x^4$

Check Your Skills

1. d. When the number under the square root symbol is negative.

There is no number you can multiply by itself to get a positive number. Therefore, a square root is undefined when the number under the square root symbol is negative.

2. b. When the denominator is zero

There is no number you can multiply by zero to get a non-zero numerator. Therefore, a fraction is undefined when its denominator is zero.

3. $\sqrt{0 - 7}$

$\left(\frac{3}{5}\right) \div 0$

The expression $\sqrt{0 - 7}$ simplifies to $\sqrt{-7}$. The square root of a negative number is undefined.

The expression $\left(\frac{3}{5}\right) \div 0$ divides by zero. Division by zero is undefined.

4. c. 4

When $x = 4$, the expression divides by zero, so it is undefined:

$$\frac{4}{4-4} = \frac{4}{0}$$

5. d. -5

When $x = -5$, the expression contains the square root of negative one, so it is undefined:

$$\sqrt{4 + (-5)} = \sqrt{-1}$$