Finding Distance on the Number Line

Connections

Have you ever…

- Had to find temperature differences below zero?

- Determined the distance between two numbers on a ruler or measuring tape?

- Found a distance for running down the street and back?

Number lines appear in many forms in our daily lives. For instance, thermometers are like number lines and are used to record body temperatures and outside temperatures. How can you find the temperature difference between two times in a given day? How can you use subtraction and absolute value to find this difference?

The **absolute value** of a number is its distance from 0 on a number line. Look at the diagram. You can see that −4 is a distance of 4 from 0, so |−4| = 4. Similarly, 3 is a distance of 3 from the zero, so |3| = 3. Distance is always positive, so absolute value is used to ensure that the distance remains positive.

Distance = 4   Distance = 3

−5 −4 −3 −2 −1 0 1 2 3 4 5
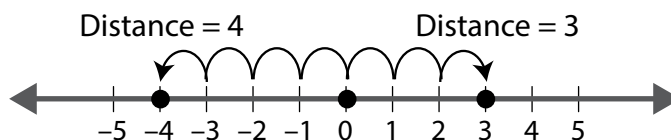
You can also use absolute value to find the **distance** between two numbers on a number line.

If a and b are two numbers on a number line, then the distance between the numbers is $|a - b|$.

When using absolute value, the order of subtraction doesn't matter. So, the distance between a and b is also $|b - a|$.

Learn It!

Distance on the Number Line

Finding the difference between two numbers on the number line is simple: just subtract. Depending on how you subtract, the difference can be positive or negative. Using absolute value ensures that the distance is positive, regardless of how you subtract.

At 1:00 p.m., Denise noticed that the outside temperature was 16° C. At 8:00 p.m. the same day, she noticed the temperature was −7° C. What two absolute value expressions can she use for the distance between these two temperatures? By how many degrees did the temperature drop from 1:00 p.m. to 8:00 p.m.?

Visualize the Problem Using a Number Line

When you use a number line, you get a better look at how far apart the numbers are.

? 1. Draw a number line to represent the problem.

Draw a horizontal number line showing the location of each number.
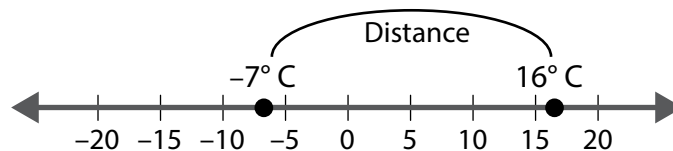
Write Expressions for the Distance

When you find a distance on a number line, you will only need to use one absolute expression to solve the problem. In this step, write both absolute value expressions to try both and experiment with which way is easier for you.

? 2. Write two absolute value expressions that represent the distance.

The expressions $|a - b|$ and $|b - a|$ will give you the same distance. Use the values from the problem: $a = -7$ and $b = 16$.

$$|-7 - 16| \ \text{and} \ |16 - (-7)|$$

Simplify the Expressions

When simplified, each expression will give you the same answer. So, the expression you use to solve a problem doesn't matter.

? **3.** Simplify each expression. Verify that they are equal.

Simplify inside the absolute value bars first, and then find the absolute value. See that the distances are the same.

$$|-7 - 16| = |-23| = 23 \qquad |16 - (-7)| = |16 + 7| = |23| = 23$$

Interpret the Distance

The question asks for the drop in temperature from 1:00 p.m. to 8:00 p.m. Does the result make sense? What is the difference in temperature?

? **4.** What was the drop in temperature from 1:00 p.m. to 8:00 p.m?

In the last step, you found that the distance between the two temperatures on the number line is 23. The question is asking about a difference in degrees, so the answer is 23 degrees. Write the answer as a sentence.

From 1:00 p.m. to 8:00 p.m., the temperature dropped 23° C.

If the the temperature started at 16 degrees and dropped to –7, it sounds reasonable that the drop is 23 degrees. You can check your results on the number line. If you look at the number line, the numbers –7 and 16 are 23 units apart from each other.

Practice It!

Answer the following questions.

★★ **1.** Shawn didn't keep track of the checks he's been writing. He had a bank account balance of –$44. He deposited some money into his bank account so that his balance was $38.

a. Use a number line to represent the problem.

b. Write an absolute value expression that can be used to find how much Shawn deposited into his account.

c. How much did Shawn deposit into his account?

★★ **2.** A diver's distance below sea level is –42 feet. The diver ascended to –19 feet below sea level.

a. Use a number line to represent the problem.

b. Write an absolute value expression that can be used to find the distance the diver ascended.

c. How many feet did the diver ascend?

3. Marissa's patient had a temperature of 102.8° F. The next day her patient had a normal temperature of 98.6° F.

    a. Use a number line to represent the problem.

    b. Write an absolute value expression that can be used to find the patient's drop in temperature.

    c. How many degrees did the patient's temperature drop?

4. What numbers are 3 units from –1 on the number line?

    a. Use a number line to represent the problem.

    b. Use the number line to identify the numbers.

    c. Write four absolute value expressions for the distance between –1 and the numbers you've identified.

Build Your Math Skills

The expressions $|a - b|$ and $|b - a|$ will always give you the same distance. You can use either one to solve a problem.

Use whichever expression is easiest for you.

5. A shark is –220 feet below sea level. What distance is the shark from sea level?

    a. Use a number line to represent the problem.

    b. Write two absolute value expressions that can be used to find the distance.

    c. Simplify one of the expressions from Part b to find the distance.

6. The expression $|82.77 - 45.03|$ can be use to find the difference in volumes of two containers of liquid, measured in liters.

    a. Write another absolute value expression that represents the difference.

    b. Use the expression you wrote to find the difference in volumes.

Check **Your Skills**

Use your knowledge of finding a distance on a number line to answer the following questions.

1. What is the distance between 15 and –20 on the number line?

    a. –5

    b. –35

    c. 35

    d. 5

2. The record low temperature in Fairbanks, Alaska for the month of January was –60° F, and the record high was 47° F. Which absolute value expression represents the difference between these temperatures?

    a. $|60 - 47|$

    b. $|47 + (-60)|$

    c. $|-60 - (-47)|$

    d. $|-60 - 47|$

3. What is the distance between –23.6 and –2.7 on the number line?

    a. 20.9

    b. –26.3

    c. 26.3

    d. –20.9

**4.** A bank account was overdrawn by $68.34. Another bad check was written bringing the balance to –$92.33. Which absolute value expression represents the amount of the check?

    a. $|68.34 - (-92.33)|$

    b. $|-92.33 - 68.34|$

    c. $|-68.34 - 92.33|$

    d. $|-68.34 - (-92.33)|$

5. The numbers m and n are two points on a number line. Which absolute value expression represents the distance between m and n?

    a. $|m + n|$

    b. $|n - m|$

    c. $|n + m|$

    d. $|n - (-m)|$

6. Which expressions represent the distance between point P and point Q? Select all that apply.
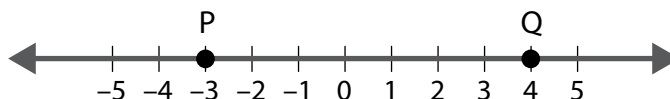
    ☐ $|4 + (-3)|$

    ☐ $|-3 + 4|$

    ☐ $|4 - (-3)|$

    ☐ $|-3 - 4|$

7. One day, the low temperature was –12° F. The temperature increased by 28° F over an 8-hour period.

Which absolute value expression can be used to find the difference between these two temperatures?

    a. $|-12 - 16|$

    b. $|-12 + 16|$

    c. $|8 - 28|$

    d. $|28 + (-12)|$

Remember the Concept
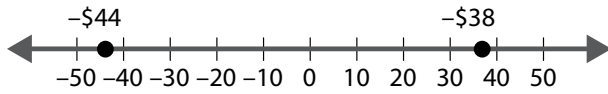
To find the distance between two points on a number line:

- Subtract

- Use the absolute value of the answer

Finding Distance on the Number Line

Distance on the Number Line

Practice It!

1a.

-$44   -$38

$$-50 \ -40 \ -30 \ -20 \ -10 \quad 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50$$

1b. $|-44 - 38|$

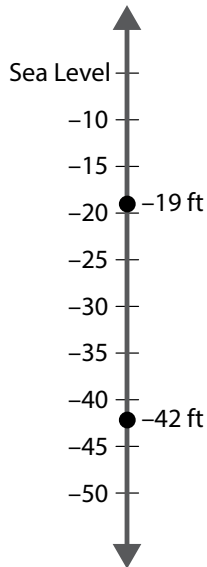Let $a = -44$ and $b = 38$ then use $|a - b|$ to represent the distance.

The expression $|38 - (-44)|$ can also be used if you let $a = 38$ and $b = -44$.

1c. $82

$|-44 - 38| = |-82| = 82$

Or, $|38 - (-44)| = |38 + 44| = |82| = 82$

2a.

Sea Level
-10
-15
-20 •−19 ft
-25
-30
-35
-40
-42 ft
-45
-50

2b. $|-42 - (-19)|$

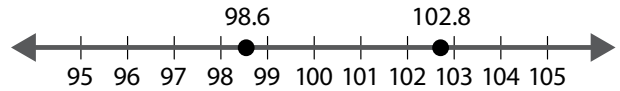Let $a = -42$ and $b = -19$, then use $|a - b|$ to represent the distance.

The expression $|-19 - (-42)|$ can also be used if you let $a = -19$ and $b = -42$.

2c. The diver ascended 23 ft.

$|-42 - (-19)| = |-42 + 19| = |-42 + 19| = |-23| = 23$

Or, $|-19 + 42| = |23| = 23$

3a.

98.6   102.8

$$95 \ 96 \ 97 \ 98 \ 99 \ 100 \ 101 \ 102 \ 103 \ 104 \ 105$$

3b. $|98.6 - 102.8|$

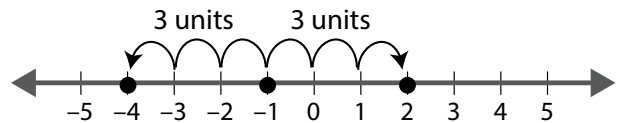Let $a = 98.6$ and $b = 102.8$, then use $|a - b|$ to represent the distance.

The expression $|102.8 - 98.6|$ can also be used if you let $a = 102.8$ and $b = 98.6$.

3c. $4.2°\,F$

$|98.6 - 102.8| = |-4.2| = 4.2$

Or, $|108.2 - 98.6| = |4.2| = 4.2$

4a.

3 units   3 units

$$-5 \ -4 \ -3 \ -2 \ -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$
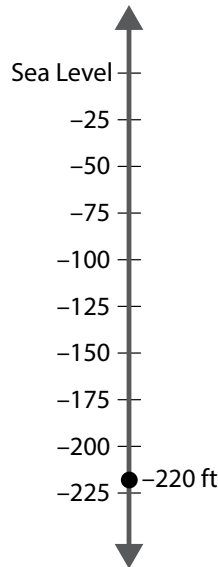
4b. -4 and 2

Count 3 units in each direction from –1.

4c. $|-4 - (-1)|$, $|-1 - 2|$, $|-1 - (-4)|$ and $|-2 - 1|$

Use the rule $|a - b|$ to write the absolute value expressions.

5a.

Sea Level
-25
-50
-75
-100
-125
-150
-175
-200
-225 •−220 ft

5b. $|0 - (-220)|$ and $|-220 - 0|$

5c. 220 ft

$|0 - (-220)| = |0 + 220| = |220| = 220$

Or, $|-220 - 0| = |-220| = |220| = 220$

6a. $|45.03 - 82.77|$

The values you use for a and b in $|a - b|$ will give the same difference.

6b. 37.74 liters

$|45.03 - 82.77| = \ |45.03 - 82.77| = |-37.74| = 37.74$

Check Your Skills

1. c. 35

Let $a = 15$ and $b = -20$ to get the distance $|15 - (-20)| = |15 + 20| = |35| = 35$.

You can also let $a = -20$ and $b = 15$ to get the distance $|-20 - 15| = |-35| = 35$.

2. d. $|-60 - 47|$

Let $a = -60$ and $b = 47$ to get the expression $|-60 - 47|$.

3. a. 20.9

Let $a = -23.6$ and b $= -2.7$ to get the distance $|-23.6 - (-2.7)| = |-23.6 + 2.7| = |-20.9| = 20.9$.

You can also let $a = -2.7$ and $b = -23.6$ to get the distance $|-2.7 - (-23.6)| = |-2.7 + 23.6| = |-20.9| = 20.9$.

4. d. $|68.34 - (-92.33)|$

Let $a = 68.34$ and $b = -92.33$ to get the expression $|68.34 - (-92.33)|$.

5. b. $|n - m|$

Let $a = n$ and $b = m$ to get the expression $|n - m|$.

6. $|4 - (-3)|$

$|-3 - 4|$

Let $a = 4$ and $b = -3$ to get the distance $|4 - (-3)|$.

Let $a = -3$ and $b = 4$ to get the distance $|-3 - 4|$.

7. a. $|-12 - 16|$

Start by finding the high temperature: $-12 + 28 = 16$.

Let $a = -12$ and $b = 16$ to get the distance $|-12 - 16|$.