Using Scientific Formulas

Connections

Have you ever…

- Calculated a time of arrival for a road trip?
- Converted between cups and tablespoons for a recipe?
- Figured out your gas mileage?

Formulas show you the relationship between things in the world. Whether it's the relationship between distance and speed, or the relationship between cups and tablespoons, formulas help you use these relationships to answer every-day questions such as "How many cups are in a quart of tea?" or "When will we arrive at Aunt Hannah's house?"

To understand formulas, you should know the following important terms.

Variable	A variable is a symbol, often a letter, that represents an unknown value.	**Example:** In the statement $x = 4y + 2$, the variables are x and y.
Constant	A constant is a number that doesn't change. It is a number or a symbol like π that stands for a specific value.	**Example:** In the statement $x = 4y + 2$, the constants are the numbers 4 and 2.
Operator	An operator is a symbol representing a mathematical operation.	**Example:** In the statement $x = 4y + 2$, the addition symbol is an operator.

Scientific formulas use variables that represent different values, such as speed or distance. In your particular circumstance, you often know the value of a variable and can change it to a constant.

Learn It!

Unpacking and Solving Formulas in Three Steps

If you've ever followed a recipe, formulas will be familiar to you. Like a recipe, a formula gives you a list of ingredients along with instructions on how to combine them. These steps will help you translate the formula's "recipe" into an understandable set of instructions.

Imagine that you have a friend in Australia. She complains of a heatwave with temperatures of 45 degrees Celsius. How hot is this in degrees Fahrenheit?

Translate the Variables into Words

Like an abbreviation, the variables of a formula represent words. Once you understand what the variables mean, you have a better understanding of what the formula is asking you to do.

?

1. The formula for converting celsius to fahrenheit is $℃ \times \frac{9}{5} + 32 = ℉$. There are two variables, ℃ and ℉. Write the formula with descriptive words instead of these variables.

Think about the variables. What numbers in the problem might they represent? Are you familiar with the symbols? This problem is pretty straightforward. The variable ℃ translates to 'degrees Celsius' and the variable ℉ translates to 'degrees Fahrenheit.' If you replace the formula's variables with the words they represent, the formula looks like:

$$\text{degrees Celcius} \times \frac{9}{5} + 32 = \text{degrees Fahrenheit}$$

As you can see, the formula's meaning is much clearer.

Plug in the Numbers

Now that you can more clearly see what the variables mean, you can plug in the numbers provided by the problem, replacing the variable with the provided value. A typical formula will give you the value for one variable and expect you to solve for the other variable.

Build Your Math Skills

No operators? No problem!

If any combination of variables or constants are shown side by side, with no operators between them, the terms are multiplied.

Example: The formula $2x = yz$ could be read as "Two times x equals y times z."

? **2.** Your friend in Australia mentioned that the temperature has been 45 degrees Celsius. Replace the appropriate variable with this value.

The temperature is provided in degrees Celsius. This is a clue that the value should be plugged into the variable that represents degrees Celsius. If you replace that variable with the provided value (45 degrees) your formula looks like this:

$$45 \times \frac{9}{5} + 32 = \text{degrees Fahrenheit}$$

The formula is now a solvable math problem.

Do the Math

For the final step, add, subtract, multiply, and divide as you would with any other math problem.

? **3.** Solve the equation. How hot is 45 degrees celsius in degrees fahrenheit?

The final problem is $45 \times \frac{9}{5} + 32 = \text{degrees fahrenheit}$. First, multiply 45 by $\frac{9}{5}$. This gives you a product of 81. Next, add 32, for a final answer of 113 degrees fahrenheit.

$$45 \times \frac{9}{5} + 32 = \text{degrees Fahrenheit}$$

$$81 + 32 = \text{degrees Fahrenheit}$$

$$113 = \text{degrees Fahrenheit}$$

That's quite a heat wave!

Review & Practice

To multiply a whole number by a fraction, multiply the whole number by the numerator:

$$45 \times 9 = 405$$

Then, divide by the denominator:

$$405 \div 5 = 81$$

Practice It!

Use scientific formulas to answer the following practice problems.

1. The Mars Curiosity Rover has a mass of 8,500 pounds, where gravitational acceleration is 1 G. Curiosity weighs considerably less on Mars, however, where gravitational acceleration is only 0.38 G.

The formula for calculating weight is:

$$W = mg$$

a. Write out the formula for calculating weight, but replace the variables with the words that they represent.

b. You want to know how much Curiosity weighs on Mars. Write out the formula for calculating weight, but replace the variables with the relevant values provided in the text.

c. Using the formula for calculating weight, solve for Curiosity's weight on Mars.

2. Density measures how much mass exists inside of an object by volume. A piece of driftwood has a mass of 25 grams and a volume of 30 cubic centimeters.

The formula for calculating density is:

$$D = m \div v$$

a. Write out the formula for calculating density, but replace the variables with the words that they represent.

b. You want to know the density of the piece of driftwood. Write out the formula for calculating density, but replace the variables with the relevant values provided in the text.

c. Using the formula for calculating density, solve for the driftwood's density, rounded to the nearest hundredth place. Provide the answer in the correct units.

Build Your Math Skills

Dividing Numbers with Different Units

Unless the problem says otherwise, when you're dividing two numbers that are given in different units, the answer is given as a ratio.

Example: Consider miles and hours. If a car drives 110 miles in two hours, the car is driving 55 *miles per hour*. The "per" here indicates a ratio.

3. Power is the rate at which energy is used or consumed over time. To complete a pull-up, Jose expends 500 joules of energy. Jose completes one pull-up every two seconds.

The formula for calculating power is:

$$P = w \div t$$

a. Write out the formula for calculating power, but replace the variables with the words that they represent.

b. You want to know the rate that Jose expends power performing pull-ups. Write out the formula for calculating power, but replace the variables with the relevant values provided in the text.

c. Using the formula for calculating power, solve for the rate that Jose expends power performing pull-ups. Provide the answer in the correct units.

4. When you first learn a formula, you will often learn what the variables represent. For example, this is the formula for acceleration:

$$a = \Delta v \div t$$

The variable Δv means change in velocity, which can be calculated by subtracting the starting velocity from the ending velocity. The variable t represents time, and the variable a represents acceleration.

a. A Porsche 918 Spyder accelerates from zero to 60 mph in 2.2 seconds. How fast does it accelerate, in miles per second squared (m/s^2)? Show your work.

b. A Porsche 911 Turbo S accelerates from zero to 60 mph at a rate of 0.4 m/s^2. How much time does it take for the 911 Turbo S to accelerate from zero to 60 mph?

5. If variables in a formula are not defined, how can you determine what a variable represents?

Build Your Math Skills

Different Units of the Same Measure

If you have different units of the same measurement, such as seconds and hours, which both measure time, convert each quantity to the measurement that you want in your final answer.

Check Your Skills

When you see this icon, you may use a calculator.

Read the problem and select the choice that best answers the question. Remember to approach each problem using the steps for unpacking and solving formulas.

1. Kinetic Energy is the energy, expressed in joules, that mass possesses when it is in motion. The formula for calculating kinetic energy is:

$$K = \frac{1}{2} mv^2$$

A vehicle weighing 2200 kilograms is traveling at 20 meters per second. How much kinetic energy does it possess?

- **a.** K = 4,400 joules
- **b.** K = 4,000 joules
- **c.** K = 44,000 joules
- **d.** K = 440,000 joules

2. Mass is how much matter is inside of an object. The formula for finding mass is:

$$m = vd$$

Which of these two objects has greater mass, and by how much?

Object A is a chunk of granite, taking up 20 cm^3 of space, with a density of 2.6 g/cm^3.

Object B is a large piece of chalk, with a volume of 48 cm^3 and a density of 1.6 g/cm^3.

- **a.** Object B is 24.8 grams more massive than Object A.
- **b.** Object A is 24.8 grams more massive than Object B.
- **c.** Object B is 26.7 grams more massive than Object A.
- **d.** Object A and Object B have the same mass.

3. Using the formula for mass, find the volume of an object with a mass of 100 grams and a density of 2.5 g/cm^3.

Build Your Math Skills

What variables do you see repeated in more than one formula?

These are common variables that you should learn and remember.

4. Momentum is the impetus gained by a moving object. The formula for momentum is:

$$p = mv$$

Which of these two objects has more momentum, and how much more does it have?

Object A is a rock with a mass of 15 kg from an erupting volcano, traveling at 762 meters per second.

Object B is a stone with a mass of 1200 kg, rolling down a hill, traveling at 10 meters per second.

a. Object A has 471 kg m/s more momentum than Object B

b. Object B is has 570 kg m/s more momentum Object A.

c. Object B has 670 kg m/s more momentum than Object A.

d. The two objects possess the same momentum.

5. A steel beam, with a volume of 760 cm^3 and a density of 7.82 g/cm^3 falls from a construction site. It hits the ground traveling 16 meters per second. What was the momentum of the steel beam at the time it impacted the ground? The answer will be given in kilogram meters per second.

Formula for mass: $m = vd$

Formula for momentum: $p = mv$

a. 950,0912 m/s

b. 371.45 kg m/s

c. 95,091.2 kg m/s

d. 1,555 kg m/s

6. A force is a push or pull upon an object. The formula for the amount of force needed to accelerate an object is:

$$F = ma$$

An object with a mass of 100 kg is accelerating at the rate of 2 m/s^2. What force (in Newtons, kilogram meters per seconds squared) is needed to accelerate the object?

Remember the Concept

- Determine what the variables of a formula mean.
- Write the formula in words.
- Substitute values for the variables.
- Solve the problem.

Scientific formulas show you the relationships between things in the world—so they're important in understanding a lot of aspects of science.

Using Scientific Formulas

Unpacking and Solving Formulas in Three Steps

Practice It!

1a. weight = mass × gravitational acceleration

The variable W stands for weight. The variable m stands for mass. The variable g stands for gravitational acceleration. Since there are no operators between m and g, you multiply the two variables.

You can relate the information given in the problem (mass, gravitational acceleration) to letters representing variables in the formula (m, g).

1b. weight = 8,500 lb x 0.38

The mass is 8,500 pounds, and the gravitational acceleration is 0.38 G.

1c. weight = 3,230 pounds

Multiply mass times gravitational acceleration.

2a. density = mass ÷ volume

The variable D stands for density. The variable m stands for mass. Notice that m also stood for mass in the previous equation. The variable v stands for volume.

2b. density = 25 g ÷ 30 cm^3

The mass is 25 grams, and the volume is 30 cubic centimeters.

2c. density = 0.83 grams per cubic centimeter

Divide mass by volume.

3a. power = work ÷ time

The variable P stands for power. The variable t stands for time. The variable w stands for work. You might have some difficulty relating w to "joules of energy," but you should be able to identify that the other variables stand for power and time. If you put "energy" instead of "work," that's acceptable.

3b. power = 500 joules ÷ 2 s

The work or energy expended is 500 joules, and the time is 2 seconds.

3c. power = 250 joules per second

Divide wok by time.

4a. acceleration = change in velocity ÷ time

acceleration $= 60\ \text{mph} \div 2.2\ \text{s}$

Change hours to seconds:

acceleration $= (60\ \text{m} \div 60\ \text{s}) \div 2.2\ \text{s}$

acceleration $= 1\ \text{m/s} \div 2.2\ \text{s}$

acceleration $= 0.\overline{45}\ \text{m/s}^2$

4b. acceleration = change in velocity ÷ time

$0.4\ \text{m/s}^2 = 60\ \text{mph} \div$ time

Change hours to seconds:

$0.4\ \text{m/s}^2 = (60\ \text{m} \div 60\ \text{s}) \div$ time

$0.4\ \text{m/s}^2 = 1\ \text{m/s} \div$ time

(time) $0.4\ \text{m/s}^2 = 1\ \text{m/s}$

time $= 1\ \text{m/s} \div 0.4\ \text{m/s}^2$

time $= 2.5\ \text{s}$

5. Answers will vary. Often, variables are represented by a letter that stands for a word, such as t for time or v for velocity. This helps you identify the meaning of the variable. If there is a formula to find a specific value, like a "formula for mass," the variable to the left of the equals sign will usually represent what the formula is meant to find. Some variables are used in many science equations, such as m for mass. At other times, a letter might be used for more than one variable in different equations. The letter v might stand for velocity or for volume. If you understand the meaning of the equation, such as understanding that density is the amount of mass fit into a certain volume, you can understand that to get density, you would divide mass by volume.

Check Your Skills

1. d. K = 440,000 joules

kinetic energy = ½ mass × velocity squared

kinetic energy $= ½(2200) \times 20^2$

kinetic energy $= 1100 \times 400$

kinetic energy $= 440{,}000$

2. a. Object B is 24.8 grams more massive than Object A.

mass = volume × density

Object A: mass $= 20 \times 2.6 = 52$

Object B: mass $= 48 \times 1.6 = 76.8$

$76.8 - 52 = 24.8$

3. 40 cm^3

mass = volume × density

$100\ g$ = volume × $2.5\ g/cm^3$

$100\ g \div 2.5\ g/cm^3$ = volume

$40\ cm^3$ = volume

4. c. Object B has 570 kg m/s more momentum than Object A.

momentum = mass × velocity

You might be tempted to think that m stands for momentum, but remember, this is a formula for momentum. The variable for momentum will be left of the equal sign. In this case the variable for momentum is p.

Object A: momentum = $15 \times 762 = 11{,}430$

Object B: momentum = $1{,}200 \times 10 = 12{,}000$

$12{,}000 - 11{,}430 = 570$

5. c. 95,091.2 kg m/s

This problem doesn't provide the mass of the steel beam. To find this value, you must use the mass formula. Once you plug in the volume and density values for the steel beam, you will find it has 5,943.2 kilograms of mass.

mass = volume × density

mass = $760 \times 7.82 = 5{,}943.2$

Next, take the formula for momentum and plug in the velocity of the steel beam, 16 meters per second, and the mass you just calculated for the steel beam: 5,943.2 kilograms. The final answer comes out to a momentum of 95,091.2 kilogram meters per second.

momentum = mass × velocity

momentum = $5{,}943.2 \times 16 = 95{,}091.2$

6. a. 200 N

force = mass × acceleration

force = $100 \times 2 = 200$